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Semileptonic decays of polarised top quarks: $V + A$ admixture and QCD corrections

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Abstract:
The semileptonic decays of polarised top quarks are analysed for a general chirality-conserving tbW vertex. We calculate double differential distributions for the charged lepton and the neutrino to order α_s in the QCD coupling. We present these QCD corrections in terms of compact parameterisations that should be useful for the future investigation of the structure of the top decay vertex on the basis of large data samples.

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So far the experimental information about the decays of top quarks is not very detailed. Data from the Tevatron are consistent with the expectation that these decays are governed by the $V - A$ charged current interactions of the standard model (SM); but because of the size of the present experimental errors sizable new physics effects can not be excluded (c.f. for instance [1]).

In view of the extremely large mass of the top quarks and the circumstance that they do not hadronise these particles are excellent probes of new interactions that may uncover at energies of a few hundred GeV. There is a vast literature on the phenomenology of such interactions in top quark decays (for overviews, see for instance [2, 3]). One possibility is a small $V + A$ admixture to the standard left-handed $t \to b$ current as is predicted for instance by $SU(2)_L \times SU(2)_R \times U(1)$ extensions of the standard model. The measured branching ratio of the radiative weak decay $b \to s\gamma$ provides a stringent constraint [4, 5, 6, 7] on the coupling strength κ_R of such a right-handed admixture: $|\kappa_R| \lesssim 0.04$. Of course, one may envisage contrived scenarios where other contributions to $b \to s\gamma$ lead to cancellations that invalidate this bound – in any case, a direct search for such a coupling in t and $\bar{t}$ decays is indispensable. Sensitive tests require polarised top quarks. At the LHC the single top production processes will yield large samples of polarised t and $\bar{t}$ quarks, and it has been estimated [8, 9] that a statistical sensitivity $\delta\kappa_R \simeq 0.06$ can be reached. From $t\bar{t}$ pair production at a high-luminosity linear e^+e^- collider a sensitivity $\delta\kappa_R \simeq 0.03$ will be feasible [10, 11, 3].

Energy and angular distributions for polarised top quark decays with a $V + A$ admixture in the tbW vertex were investigated in a number of articles, including [12, 13, 14, 15]. In ref. [13] semileptonic t decays were analysed and it was pointed out that the polarisation-dependent part of the neutrino distribution (which can be determined experimentally by measuring the missing momentum) is very sensitive to κ_R. The order α_s QCD corrections for the $V - A$ part of the charged current [16, 17] were incorporated in [13]. In view of the above-mentioned expected sensitivity to κ_R at future colliders one should note that taking into account QCD corrections in future data analysis is mandatory, because they can mimic a small $V + A$ admixture.

In this paper we extend the work of [13] in that we compute the order α_s QCD corrections to both the $V - A$ and $V + A$ Born contributions to $t \to b\ell\nu_\ell$. Moreover we take the finite width of the intermediate W boson and the non-zero mass of the b quark into account. The latter is important in the case of a small $V + A$ admixture, because terms linear in κ_R in the lepton distributions, which we compute below, require a chirality flip of the b quark. For small κ_R, say $|\kappa_R| < 0.1$, the terms proportional to $\kappa_R m_b/m_t$ are of the same order of magnitude as the contributions proportional to κ_R^2. Furthermore we compute a T-odd triple correlation which is generated by a non-standard CP-violating phase that may be present in the general chirality-conserving tbW vertex.

We consider the semileptonic decay of spin-polarised top quarks to order α_s in

the QCD coupling, which amounts to studying the reactions

$$\begin{aligned} t &\to b(p_b) + \ell^+(p_\ell) + \nu_\ell(p_\nu)\,, \\ t &\to b(p_b) + \ell^+(p_\ell) + \nu_\ell(p_\nu) + g(p_g)\,. \end{aligned} \tag{1}$$

All momenta refer to the rest frame of the top quark. As far as the tbW vertex is concerned we use the generalised interaction

$$\mathcal{L}_{tb} = -\frac{g}{2\sqrt{2}} V_{tb}\, \bar{t}\gamma^\mu(\alpha - \beta\gamma_5) b\, W_\mu \quad + \quad \text{h.c.}\,, \tag{2}$$

where α, β are complex couplings. A right-handed admixture to the $V-A$ charged current interaction of the SM ($\alpha = \beta = 1$) will be parameterised by choosing $\alpha = 1 + \kappa_R$, $\beta = 1 - \kappa_R$. In the calculations below we neglect the lepton masses but take the mass of the b quark and the finite width of the intermediate W boson into account.

In the calculation of the differential decay distributions for (1) we have performed the quark wave function renormalisations that remove the ultraviolet divergencies in the on-shell scheme. The infrared divergencies are canceled using a standard phase space slicing procedure. The phase space of the four-particle final state in the reaction $t \to b\ell\nu_\ell g$ is split into two disjoint regions where the (scaled) energy $x_g = 2E_g/m_t$ of the gluon is smaller and larger than an arbitrary, but small separation parameter x_{min}, respectively: $1 = \Theta(x_{min} - x_g) + \Theta(x_g - x_{min})$. Integrating the respective squared matrix element over the phase space of the soft gluon ($x_g \le x_{min}$) and adding the result to the order α_s squared matrix element for the three particle final state $b\ell\nu_\ell$ yields the infrared finite differential decay distribution $d\Gamma_{B+V+soft}$ (B = Born, V = virtual) and $d\Gamma_{hard}$, which describes radiation of "resolved" gluons ($x_g > x_{min}$).

For checks of possible deviations from the $V-A$ law in the tbW vertex useful observables are the double differential energy-angle distributions $d\Gamma/dx_i d\cos\theta_i$ ($i = \ell, \nu$) for the charged lepton and for the neutrino, where $x_i = 2E_i/m_t$ and θ_i is the angle between the three-momentum of the lepton i and the unit vector $\hat{\mathbf{s}}$ that specifies the polarisation direction of the ensemble of top quarks in the t rest frame. The degree of polarisation is denoted by $S = |\mathbf{s}|$. Adding the contributions from $d\Gamma_{B+V+soft}$ and from $d\Gamma_{hard}$, these distributions can be put into the following form:

$$\frac{d\Gamma}{dx_\ell d\cos\theta_\ell} = |V_{tb}|^2 \frac{g^4 m_t}{8} \left[(F_0^{\ell n} + C_F \frac{\alpha_s}{\pi} F_1^{\ell n}) + (F_0^{\ell s} + C_F \frac{\alpha_s}{\pi} F_1^{\ell s}) S\, \cos\theta_\ell \right], \tag{3}$$

$$\frac{d\Gamma}{dx_\nu d\cos\theta_\nu} = |V_{tb}|^2 \frac{g^4 m_t}{8} \left[(F_0^{\nu n} + C_F \frac{\alpha_s}{\pi} F_1^{\nu n}) + (F_0^{\nu s} + C_F \frac{\alpha_s}{\pi} F_1^{\nu s}) S\, \cos\theta_\nu \right], \tag{4}$$

where $C_F = 4/3$ and $0 \le x_\ell, x_\nu \le (m_t^2 - m_b^2)/m_t^2$. The corresponding distributions for the anti-top quark are obtained by changing the sign of the term proportional

to $\cos\theta_i$. From these distributions the corresponding 2×2 top decay spin density matrices can be extracted in straightforward fashion[1].

The F_a^{ij} ($i=\ell,\nu$, $j=n,s$, and $a=0,1$) which are functions of x_ℓ and x_ν, respectively, can be decomposed according to the contributions from the vector and axial vector couplings α,β :

$$F_a^{ij} = |\alpha|^2 F_{a,\alpha}^{ij} + \mathrm{Re}\,(\alpha^*\beta) F_{a,\alpha\beta}^{ij} + |\beta|^2 F_{a,\beta}^{ij}\,. \tag{5}$$

The following relations hold between the functions that appear in the charged lepton distribution and those of the corresponding neutrino distribution:

$$\begin{aligned} F_{a,\alpha}^{\ell n} &= F_{a,\alpha}^{\nu n}\,, \qquad F_{a,\beta}^{\ell n} = F_{a,\beta}^{\nu n}\,, \qquad F_{a,\alpha\beta}^{\ell n} = -\,F_{a,\alpha\beta}^{\nu n}\,, \\ F_{a,\alpha}^{\ell s} &= -\,F_{a,\alpha}^{\nu s}\,, \qquad F_{a,\beta}^{\ell s} = -\,F_{a,\beta}^{\nu s}\,, \qquad F_{a,\alpha\beta}^{\ell s} = F_{a,\alpha\beta}^{\nu s}\,, \end{aligned} \tag{6}$$

where $a=0,1$. For the sake of presenting compact expressions we write down the lowest order functions of the charged lepton distribution in terms of one-dimensional integrals ($r=\alpha,\beta,\alpha\beta$):

$$F_{0,r}^{\ell j} = \frac{1}{512\pi^3}\int_{x_{b,min}}^{x_{b,max}} dx_b\, D_W(x_b)\tilde{F}_{0,r}^{\ell j}(x_\ell,x_b)\,, \tag{7}$$

where $D_W = ((1-x_b-\hat{m}_W^2+\hat{m}_b^2)^2+\hat{m}_W^2\hat{\Gamma}_W^2)^{-1}$, $\hat{m}_W = m_W/m_t$, $\hat{\Gamma}_W = \Gamma_W/m_t$, $\hat{m}_b = m_b/m_t$, $x_{b,min} = (1-x_\ell+\hat{m}_b^2/(1-x_\ell))$, $x_{b,max} = (1+\hat{m}_b^2)$, and

$$\begin{aligned} \tilde{F}_{0,\alpha}^{\ell n} &= -2\hat{m}_b^3+\hat{m}_b^2(x_b-2)+2\hat{m}_b(x_b-1)-x_b^2+x_b(3-2x_\ell)-2(x_\ell-1)^2\,, \\ \tilde{F}_{0,\alpha\beta}^{ln} &= 2(1+\hat{m}_b^2-x_b)(2-x_b-2x_\ell)\,, \\ \tilde{F}_{0,\beta}^{ln} &= 2\hat{m}_b^3+\hat{m}_b^2(x_b-2)-2\hat{m}_b(x_b-1)-x_b^2+x_b(3-2x_\ell)-2(x_\ell-1)^2\,, \\ \tilde{F}_{0,\alpha}^{ls} &= -(1+\hat{m}_b^2-x_b)(2\hat{m}_b^2+x_b(x_\ell-2)+2(x_\ell-1)^2+2\hat{m}_b x_\ell)\,, \\ \tilde{F}_{0,\alpha\beta}^{ls} &= 4(1+\hat{m}_b^2-x_b)^2+2(1-\hat{m}_b^2-x_b)(x_b-4)x_\ell-4(x_b-2)x_\ell^2-4x_\ell^3\,, \\ \tilde{F}_{0,\beta}^{ls} &= -(1+\hat{m}_b^2-x_b)(2\hat{m}_b^2+x_b(x_\ell-2)+2(x_\ell-1)^2-2\hat{m}_b x_\ell)\,. \end{aligned} \tag{8}$$

The corresponding functions for the neutrino distribution are obtained with the relations (6). Using $\alpha,\beta = 1\pm\kappa_R$ it is easy to see that those terms in the Born distributions which are linear in κ_R are accompanied by a factor $\hat{m}_b$ (or $\hat{m}_b^3$).

The functions $F_{1,\alpha}^{ij}, F_{1,\alpha\beta}^{ij}$, and $F_{1,\beta}^{ij}$ which are induced by the order α_s corrections are shown in Figs. 1 and 2. We use $m_t = 174.3$ GeV, $m_b = 4.8$ GeV, $m_W = 80.42$ GeV, and $\Gamma_W = 2.12$ GeV. For SM couplings $\alpha=\beta=1$ and for the narrow width approximation $\hat{\Gamma}_W\to 0$ we can numerically compare our results with those of [17] and we find agreement.

The size of the QCD corrections is typically 6 to 8 percent for $x_i < 0.9$ and somewhat larger at the upper end of the spectrum. For $x_i\to 1$ the curves show the

[1]These density matrices enter the calculations of the (differential) cross sections that describe t and/or $\bar{t}$ production and decay in the on-shell approximation.

	$F^{\ell n}_{1,\alpha}$	$F^{\ell n}_{1,\beta}$	$F^{\ell n}_{1,\alpha\beta}$	$F^{\ell s}_{1,\alpha}$	$F^{\ell s}_{1,\beta}$	$F^{\ell s}_{1,\alpha\beta}$
$A_0[10^{-7}]$	−8.06	−10.54	−14.37	−7.55	−9.86	−15.25
$A_1[10^{-5}]$	5.50	5.50	4.96	5.24	5.24	5.18
B_0	−0.2853	−0.3087	−1.6414	−1.1058	−1.2267	−1.6760
B_1	3.3781	3.6545	22.6920	17.7909	19.7949	23.4286
B_2	−16.3481	−17.902	−131.1934	−122.4657	−136.8286	−136.6096
B_3	40.0362	44.3232	408.7910	467.4425	523.4506	428.1320
B_4	−53.1628	−59.4165	−741.6436	−1083.4018	−1213.9973	−781.1829
B_5	36.4636	41.0886	784.5948	1563.7047	1751.0343	831.2825
B_6	−10.1118	−11.4767	−448.9931	−1374.8811	−1536.9986	−478.5832
B_7			107.4585	674.4979	752.1898	115.2665
B_8				−141.6121	−157.4561	

Table 1: The coefficients which determine the fits (9) to the order α_s QCD contributions to the charged lepton distribution (3). A_0 and A_1 are given in units of 10^{-7} and 10^{-5}, respectively.

developoment of the well-known logarithmic singulariy [16] which can be removed by exponentiation. Rather than presenting the exact analytical formulae for the functions $F^{ij}_{1,\alpha}, F^{ij}_{1,\alpha\beta}$, and $F^{ij}_{1,\beta}$, which are quite lengthy, we give them in terms of simple parameterisations determined by fits. We required the fits to be better than 4% in the range $0 \leq x_\ell, x_\nu \leq 0.98$. This requirement can be satisfied by using a rational function for $x_\ell, x_\nu \leq 0.218$ and a polynomial of at most 8th order for $x_\ell, x_\nu > 0.218$. This number is the location of the peaks in Figs. 1 and 2 which correspond to the maximum of D_W and the shape of the functions of F_1 around this peak arise as follows. The contributions to F_1 from $d\Gamma_{V+soft}$ and $d\Gamma_{hard}$ have opposite signs. The virtual and soft corrections start growing steeply at a slightly lower value of x_i than the term from "hard" gluon radiation and the former level off slightly earlier, at $x_i \simeq 0.218$, where the increase of $d\Gamma_{hard}$ is still rather steep. We have checked numerically that this does not depend on the choice of the (small) separation parameter x_{min}.

In the fits we use the generic form

$$F_1 = \Theta(0.218 - x)\frac{A_0}{(x-0.218)^2 + A_1} + \Theta(x - 0.218)\sum_{k=0}^{8} B_k x^k\,, \tag{9}$$

where $\Theta(x)$ denotes the step function. The coefficients A_k, B_k that specify the respective functions are given in Table 1. The corresponding functions for the neutrino distribution are obtained with the help of (6).

The polarisation dependent part of the neutrino distribution, $F^{\nu s} \equiv F_0^{\nu s} + C_F \frac{\alpha_s}{\pi} F_1^{\nu s}$, is the term in the above distributions that is most sensitive to a coupling κ_R [13], especially for neutrino energies $x_\nu \lesssim 0.6$. For instance, for $\kappa_R = 0.1$

the function $F^{\nu s}$ deviates from its SM value by about 4% for $0.25 \lesssim x_\nu \lesssim 0.6$, and for $x_\nu \lesssim 0.25$ the deviations are even larger. In order to obtain maximal sensitivity to κ_R in future data analyses one may use likelihood functions that can be derived [10] from (3) and (4).

Finally we consider the possibility that $\mathrm{Im}(\alpha^*\beta) \neq 0$. This happens in a natural way in $SU(2)_L \times SU(2)_R \times U(1)$ extensions of the SM, where the Higgs sector is sufficiently complicated to allow for a CP-violating phase in the $L-R$ gauge boson mixing matrix already at tree level. After diagonalisation of this mass matrix this phase is transported to the charged current interaction (2). An observable to check for such a phase is the expectation value of the T-odd triple product $\mathcal{O} = \hat{\mathbf{p}}_\ell \cdot (\hat{\mathbf{p}}_b \times \hat{\mathbf{s}})$, where $\hat{\mathbf{p}}_{b,\ell}$ are the directions of flight of the b quark and the lepton, respectively, in the top quark rest frame. (At this order in the QCD coupling the decay amplitude has no absorptive part that could generate $< \mathcal{O} > \neq 0$.) We have to order α_s:

$$< \mathcal{O} >= \frac{1}{\Gamma_{SL}} \Big(\int \mathcal{O}\, d\Gamma_{B+V+soft} \;+\; \int \mathcal{O}\, d\Gamma_{hard} \Big)\,, \tag{10}$$

where Γ_{SL} denotes the order α_s partial decay width for (1). Because the deviations of the moduli of the parameters α and β from their SM values are not expected to be large we use Γ_{SL}^{SM} for the normalisation in (10). We use $\Gamma_{SL}^{SM} = 0.168$ GeV and 0.153 GeV to lowest order and to order α_s, respectively. We get $< \mathcal{O} >= c\, \mathrm{Im}(\alpha^*\beta)\, S$, where $c = 0.0050$ at tree level and $c = 0.0042$ including the order α_s corrections. This implies that the effect is too small in order to obtain an interesting sensitivity to $\mathrm{Im}(\alpha^*\beta)$ (i.e. at the percent level) from semileptonic top quark decays.

In summary we have computed, for a tbW vertex with left- and right-handed components, double differential lepton distributions to order α_s for polarised semileptonic top decay, and we presented the QCD corrections in terms of compact parameterisations. These formulae should be a useful module in the theoretical description of t and/or $\bar{t}$ production and decay at next-to-leading order in α_s, including non-standard top couplings.

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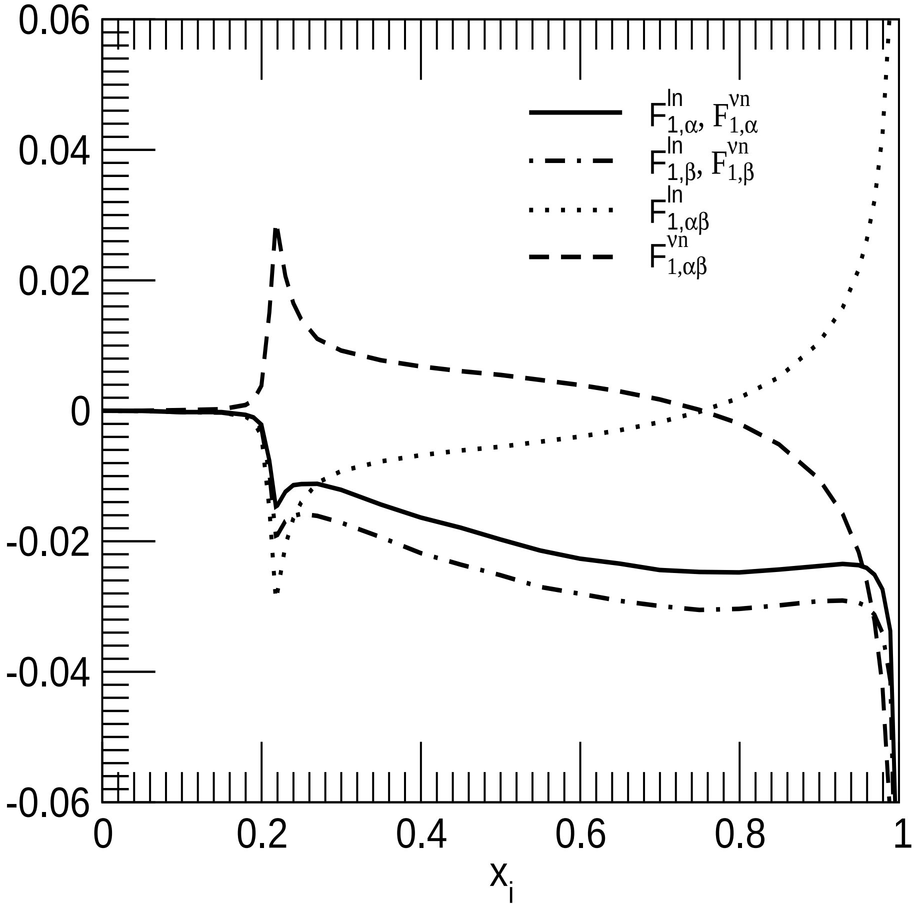
FIGURE CAPTIONS

Fig. 1. The order α_s contributions to the polarisation independent part of the lepton distributions (3), (4).
Fig. 2. The order α_s contributions to the polarisation dependent part of the lepton distributions (3), (4).



0.06
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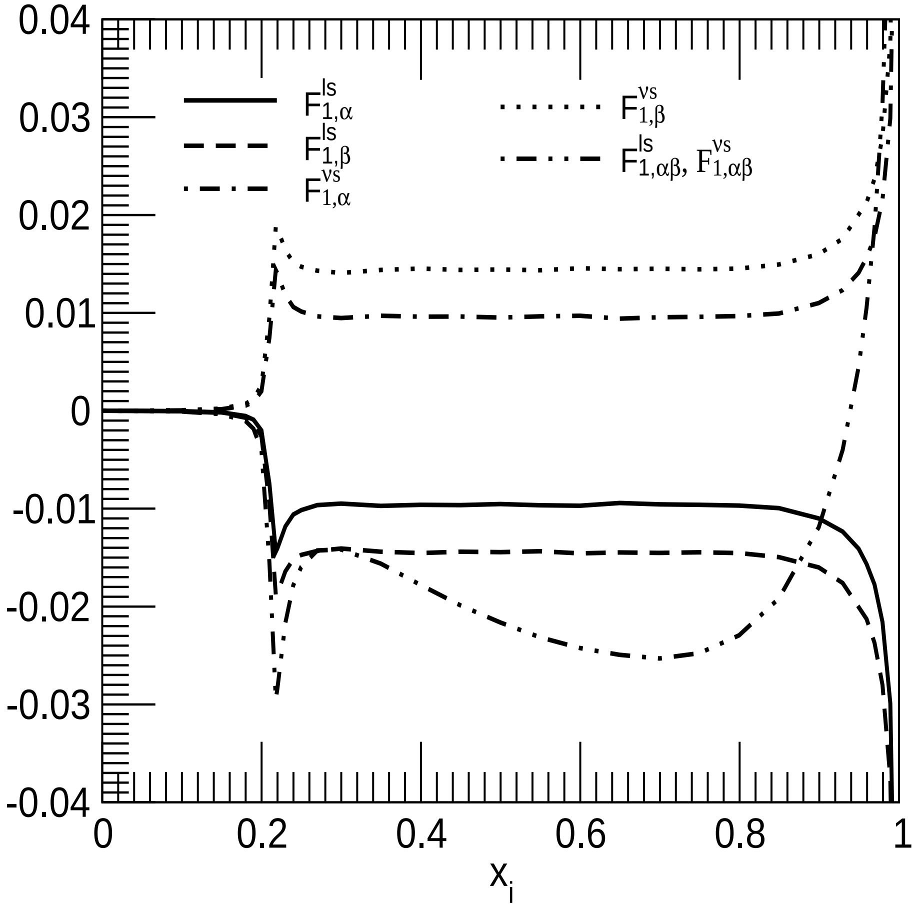
1
x_i
$F^{ln}_{1,\alpha}$, $F^{\nu n}_{1,\alpha}$
$F^{ln}_{1,\beta}$, $F^{\nu n}_{1,\beta}$
$F^{ln}_{1,\alpha\beta}$
$F^{\nu n}_{1,\alpha\beta}$

Figure 1:


0.04
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1
x_i
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$F^{ls}_{1,\beta}$
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$F^{vs}_{1,\beta}$
$F^{ls}_{1,\alpha\beta}$, $F^{vs}_{1,\alpha\beta}$

Figure 2: